Analysis Summer HW problem Set: Compiled 2017

Students bridging up to **Analysis** from Alg2/TrigA will need to learn a few topics on their own this summer. These are topics that other Analysis students learned in Alg2/TrigH. The following are links to videos for you to learn. At the end is a problem set, and solutions.

Polar Coordinates:
http://tutorial.math.lamar.edu/Classes/CalcII/PolarCoordinates.aspx
(through examples 1 and 2 only)

Vector Basics:
http://tutorial.math.lamar.edu/Classes/CalcII/Vectors_Basics.aspx

Vector Arithmatic:
http://tutorial.math.lamar.edu/Classes/CalcII/VectorArithmetic.aspx

Arithmetic and Geometric Sequences:
http://www.purplemath.com/modules/series3.htm

Arithmetic Series: http://www.mathwords.com/a/arithmetic_series.htm

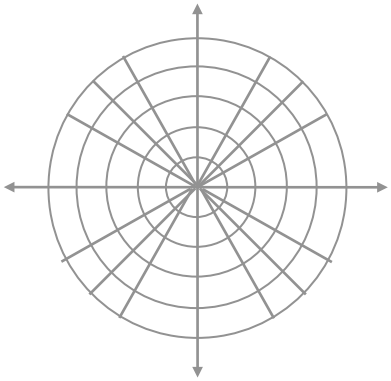
Geometric Series: http://www.mathwords.com/g/geometric_series.htm

Polar:

1. Consider the following points:

$A:(2,\frac{11\pi}{6})\quad B:(-3,\frac{-\pi}{4})\quad C:(0,2)$

a) Plot and **label** the 3 points. [3]



b) Rewrite point "A" using a negative angle and negative radius .

_____ [2]

c) Convert point B to rectangular coordinates. [2]

_____

2) Convert (-5, 4) to polar coordinates. Since you aren't allowed a calculator leave the angle in "exact terms", in other words, what you would type into your calculator.

(r,θ)=_____[3]

3. Convert the equation to rectangular form. $r = \dfrac{5}{\cos\theta - \sin\theta}$ _____

Vectors:

1. Let P = (2, 4), Q = (1, 4), R = (-2, -1) and S = (-3, -1).
   (a) Find the magnitude of $\overrightarrow{QR}$.

   _____ [
   (*exact answer*)

   (b) Show algebraically that $\overrightarrow{RP}$ and $\overrightarrow{SQ}$ are equivalent *[3 pts]*

   (c) Find point W such that $\overrightarrow{PW} = \langle -1,6 \rangle$

   _____

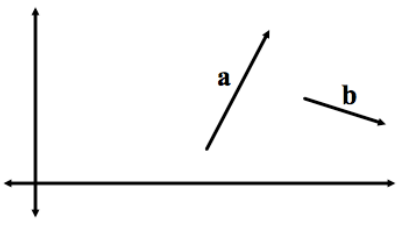
2. Let **u** = $\langle -2,-5 \rangle$, **w** = $\langle 1,-4 \rangle$, and **v** = $\langle -3,6 \rangle$
   (a) Find the component form of the vector $\frac{1}{3}\mathbf{v} - 2\mathbf{u} + \mathbf{w}$. _____ *[2 pts]*

   (b) Find the direction angle of **w**. _____ *[2pts]*
   (rounded to hundredths place)

3. Find a unit vector in the direction of **v** = **-i** + 3**j**.

   _____ *[2pts]*
   (*exact answer*)

4. Given vectors **a** and **b** below, geometrically show the vector addition of **a** + **b**. *[2 pts]*



5. An airplane flying on a bearing of 170° at 300 mph encounters a wind blowing on a bearing of 55° at 25 mph. Draw a vector diagram of the scenario. Then calculate the component forms of the velocity vectors for the plane and the wind. Then find the actual ground speed and bearing of the plane *(after taking into account the wind)*. Write your answers in the blanks provided. Make sure all your work is **neatly** shown! ***Do not round until the final answer. Round your final answers to the nearest tenth.***

Vector Diagram: *[2 pts]*

Plane's velocity vector: *[2 pts]*_____

Wind's velocity vector: *[2 pts]*_____

Actual ground speed: *[2 pts]*_____

Actual bearing: *[2 pts]*_____

Sequences and Series:

1. Write the first 4 terms and the 40th term of each sequence.

a) $a_n = \dfrac{n^2-1}{n+1}$

b) geometric sequence with $a_1 = 3$, $d = 1/3$

c) sequence defined recursively as $v_1 = -3$, $v_2 = 1$, and $v_k = v_{k-2} + v_{k-1}$, for $k \geq 3$

2. Write the first three terms of the sequence and determine whether each sequence converges. If the sequence converges, to what does it converge?

a) $\left\{\dfrac{5+4n-9n^2}{n^3+n^2-n}\right\}$ b) $\left\{\dfrac{10^n}{n!}\right\}$ c) $\left\{\dfrac{n^3+n}{n^2+100}\right\}$

3. Find a) first term b) common difference/ratio c) recursive rule d) explicit rule

i) -5, -1, 3, 7, …

ii) In a geometric sequence, the fourth term is –192 and the ninth term is 196,608.

4. Derive the formula for the sum of a finite arithmetic sequence $a_1, a_2, \ldots, a_n$

5. Write the sum using summation notation:

a) $6 - 12 + 24 - 48 + \ldots$

b) $5 - 15 + 45 - 135 + \ldots - 10{,}935$

c) $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \ldots + \frac{1}{10}$

d) $10 + 7 + 4 + \ldots - 20$

6. Determine whether the sequence is arithmetic or geometric. Then find the sum of the finite series.

a) 2, 4, 6, 8, … , 70

b) -3, 3/2, -3/4, … ; $n = 15$

7. A pendulum swings an initial arc length of 18 inches. On each swing after the first, the arc length is 0.98 of the previous swing.

a. Write an explicit rule for the length of the *n*th swing (assume the first swing is 18 inches)

b. What is the length of the 10^{th} swing?

c. What is the total length that the pendulum will have swung after 15 swings?

d. When it stops, what total distance will the pendulum have swung?

8. An auditorium has rows of seats. The first row has 20 seats. The number of seats in each row is two more than the previous row.

a. Write a recursive rule for the number of seats in any given row

b. If the last row has 42 seats, how many rows does the auditorium have? How many total seats are there in the auditorium?

9. Find the first, second, third, and 10th partial sums (calculator okay). Then decide if the series converges or diverges as *n* approaches infinity.

a) $\sum_{k=1}^{n}(5k+3)$

b) $5\sum_{k=1}^{n}\left(\frac{2}{7}\right)^{k}$

c) $\sum_{k=1}^{n}\left(k^4-(k-1)^4\right)$

10. For each of the infinite series below,

(a) decide whether the terms (a_n) of the series converge to a number (if so, give the number), and

(b) use a test or give a reason to prove that the series converges or diverges.

i. $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^n$

ii. $\frac{1}{48}+\frac{1}{16}+\frac{3}{16}+\frac{9}{16}+\ldots$

iii. $\sum_{n=1}^{\infty} \frac{1}{2n}$

iv. $\frac{3}{5}+\frac{4}{6}+\frac{5}{7}+\frac{6}{8}+\ldots$

v. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

vi. $\sum_{n=1}^{\infty} (5-2n)$

vii. $\sum_{k=0}^{\infty} \frac{1}{2^k+1}$

vi. $\sum_{k=1}^{\infty} \frac{1}{(k+1)^2}$